AME 500A
Problem Set 9
Fall 2007

The first **four** problems deal with the example on the adjoint operator discussed in class. Refer to the handout. The use of a calculator or computer program is recommended to reduce the tedium of algebra.

PROBLEM 1
Write down the vectors of $B_{dual} = \{\hat{\mathbf{e}}_1, ..., \hat{\mathbf{e}}_6\}$ as linear combinations of the $\mathbf{e}'s$. Verify that $\mathbf{e}_1 \cdot \hat{\mathbf{e}}_1 = 1, \mathbf{e}_1 \cdot \hat{\mathbf{e}}_2 = 0$ and $\mathbf{e}_2 \cdot \hat{\mathbf{e}}_2 = 1$. Show that $(B_{dual})_{dual} = B$.

PROBLEM 2
Find null spaces $Null((L-1I)^3), Null((L-2I)^2)$ and $Null((L-(3+i)I))$ by solving the matrix versions of the corresponding homogeneous equations. Assume that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, $\{\mathbf{v}_4, \mathbf{v}_5\}$ and $\{\mathbf{v}_6\}$ span these spaces, respectively. (Note: your $\mathbf{v}'s$ may be different than those on the handout. Why?)

Repeat the corresponding problem for the adjoint operator. Assume that $\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$, $\{\mathbf{w}_4, \mathbf{w}_5\}$ and $\{\mathbf{w}_6\}$ span the null spaces, respectively.

Note that in both instances the direct sum of these null spaces is the underlying vector space, $\mathbb{C}^6$. This is the primary decomposition theorem; the null spaces above come from the factors of the minimal polynomials of L and L^{adj}.

PROBLEM 3
Obtain the matrix of L relative to basis $B_1 = \{\mathbf{v}_1, ..., \mathbf{v}_6\}$. Repeat for L^{adj} relative to basis $B_2 = \{\mathbf{w}_1, ..., \mathbf{w}_6\}$. What is the form of $[L]_{B_1}$? What is the form of $[L^{adj}]_{B_2}$? At this point we are one easy step away from the Jordan forms of these matrices, but we are not going there.

Are these two matrices related by transposition and complex conjugation? What happened?

PROBLEM 4
Obtain the inner product matrix, $\hat{\Gamma}$, in the basis B_{dual} (i.e., the (i, j) element of this matrix is $\hat{\mathbf{e}}_i \cdot \hat{\mathbf{e}}_j$.)

PROBLEM 5
Consider $\mathbb{C}^6$ with basis $B_{\perp} = \{\mathbf{u}_1, \mathbf{u}_2, ..., \mathbf{u}_6\}$. The basis is orthonormal, $\mathbf{u}_i \cdot \mathbf{u}_j = \delta_{ij}$. Let $S = \text{Span}[\{\mathbf{u}_1 + i\mathbf{u}_2, \mathbf{u}_2 + 2\mathbf{u}_4 + \mathbf{u}_5, i\mathbf{u}_2 + 2\mathbf{u}_3 + \mathbf{u}_4\}]$. Find the orthogonal complement $S^{\perp}$.