AME 500A
Advanced Engineering Analysis
Fall 2007

LIST OF TOPICS

Physical Vector Calculus

The need for vectors, vectors as arrows in physical space, vector algebra, Cartesian coordinate systems, summation convention, inner (dot) product, norm (length), vector (cross) product, rotation of axes, curves and surfaces, line, surface and volume integrals, scalar and vector fields, gradient, divergence and curl, Gauss and Stokes theorems, vector identities, curvilinear coordinate systems.

Much (~80%) of the material above should be a review. This material is the mathematical backbone of most engineering subjects that you will be studying, such as fluid mechanics, solid mechanics and dynamics.

Linear Algebra

The need for linear algebra, the building blocks of linear algebra (complex numbers, vectors and linear functions).

Complex numbers: a very quick review as needed.

Vectors: vector space, linear combinations, linear independence/dependence, spanning sets, basis, dimension, subspace, sum of subspaces.

Linear Operators or Functions: basic ideas (domain, image set, range, into, onto, one-to-one, invertibility), function of a function (composition), linearity, inverse, null space, linear functions as matrices, matrix algebra and matrix operations, linear equations, solution of linear equations in matrix form (row operations and echelon form), solvability of equations, determinants, invariant subspaces, eigenproblem ($\lambda -$ multiplicities), polynomial operators (characteristic, minimal), inner product and orthogonality, orthogonal subspaces, solvability of equations (when inner product is available), the adjoint operator, the eigenproblem for the adjoint, biorthogonality, self-adjointness, the eigenproblem for self-adjoint operators, Green's function.

About (~60%) of the material will be new. You have seen most of the coverage on matrices. But I want to emphasize the fact that matrices represent linear operators and pursue an avenue of presentation whereby the **key ideas** of the subject directly go over to any linear operator, be it differential, integral, algebraic or some combination of these.

Engineering is full of "linear systems." Linear algebra is the mathematical backbone by which such systems can be solved and understood. Linear algebra is **not** simply matrix algebra. For example, the eigenvalue problem that we see for matrices is precisely the "same" as the eigenvalue problem that we see for ODE's or PDE's. This is the message that I wish to convey, although the problem for ODE's or PDE's is mathematically more subtle because the underlying vector space has infinite dimension (so convergence issues arise.) But if we truncate the infinite dimension of the vector space to a finite dimension (as we always do on the computer in a numerical calculation) the material presented in this course is directly applicable to any linear system.

Ordinary Differential Equations (ODE)

Application of linear algebra to the initial value problem for a system of ODE's, fundamental matrix and its use, second order ODE, inner product, adjoint equation, self-adjointness, eigenproblems, Green's function, eigentransforms.

About (~50%) of the material will be new. The material on second order ODE's leads to Fourier series and the Sturm-Liouville problem.

This is a very ambitious list of topics. I'm assuming that you know your undergraduate mathematics on which I can build. I'm also assuming that you want to learn some new things – thereby advancing your knowledge by absorbing advanced topics, as the course title implies.

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