12.6

d) $f = \sin \frac{1}{z}$

$f' = -\frac{1}{z^2} \cos \frac{1}{z}$

f' is analytic everywhere except $z = 0$

f) ~~12.~~ $f = x + y + i(\sin x + \cos y) = u + iv$

$u_x = 1 \qquad v_x = \cos x$

$u_y = 1 \qquad v_y = -\sin y$

$f' = u_x + i v_x$ where f is differentiable
$= 1 + i \cos x$

f is differentiable only at $y \in \{\ldots, -\frac{5\pi}{2}, -\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}, \ldots\}$
$x \in \{\pm\pi, \pm 3\pi, \ldots\}$

f is analytic nowhere

12.12

a)

$$e^{z_1} e^{z_2} = e^{x_1}\left(\cos y_1 + i \sin y_1\right) e^{x_2}\left(\cos y_2 + i \sin y_2\right)$$

$$= e^{x_1} e^{x_2}\left(\cos y_1 \cos y_2 - \sin y_1 \sin y_2 + i\left(\sin y_1 \cos y_2 + \cos y_1 \sin y_2\right)\right)$$

$$= e^{x_1+x_2}\left(\cos(y_1+y_2) + i \sin(y_1+y_2)\right)$$

$$= e^{x_1+x_2+i(y_1+y_2)} = e^{z_1+z_2}$$

b)

$$\cos(x+iy) = \cos z = \frac{1}{2}\left(e^{iz} + e^{-iz}\right) = \frac{1}{2} e^{-y+ix} + \frac{1}{2} e^{y-ix}$$

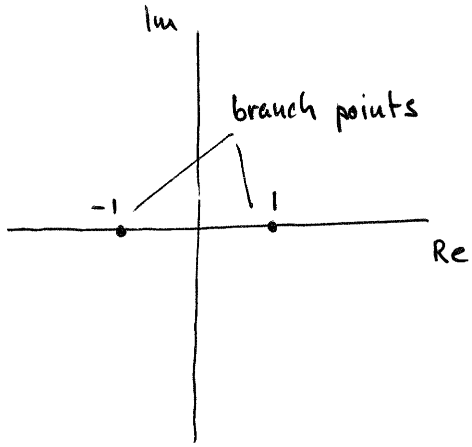
$$= \frac{1}{2}\left(e^{-y}(\cos x + i \sin(+x)) + e^{y}(\cos(-x) + i \sin(-x))\right)$$

$$= \cos x \, \frac{1}{2}\left(e^{-y} + e^{y}\right) + i \sin x \, \frac{1}{2}\left(e^{-y} - e^{y}\right)$$

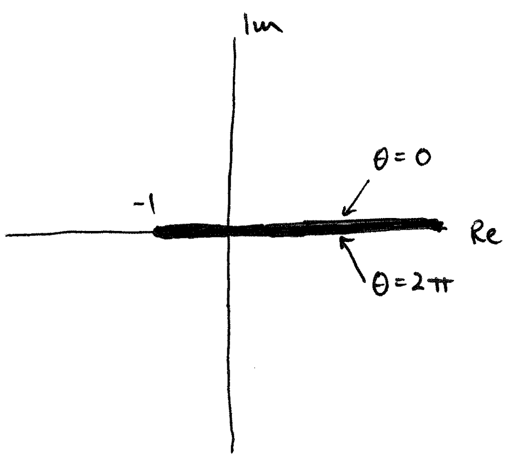
$$= \cos x \cosh y - i \sin x \sinh y$$

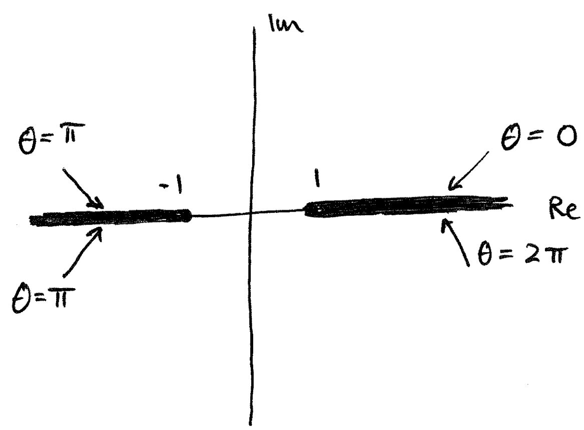
12.15

g)

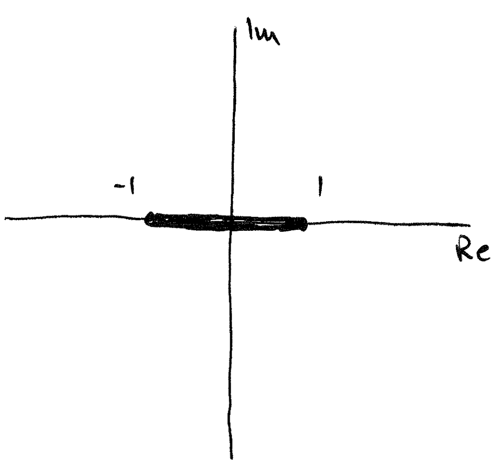


possible branch cuts

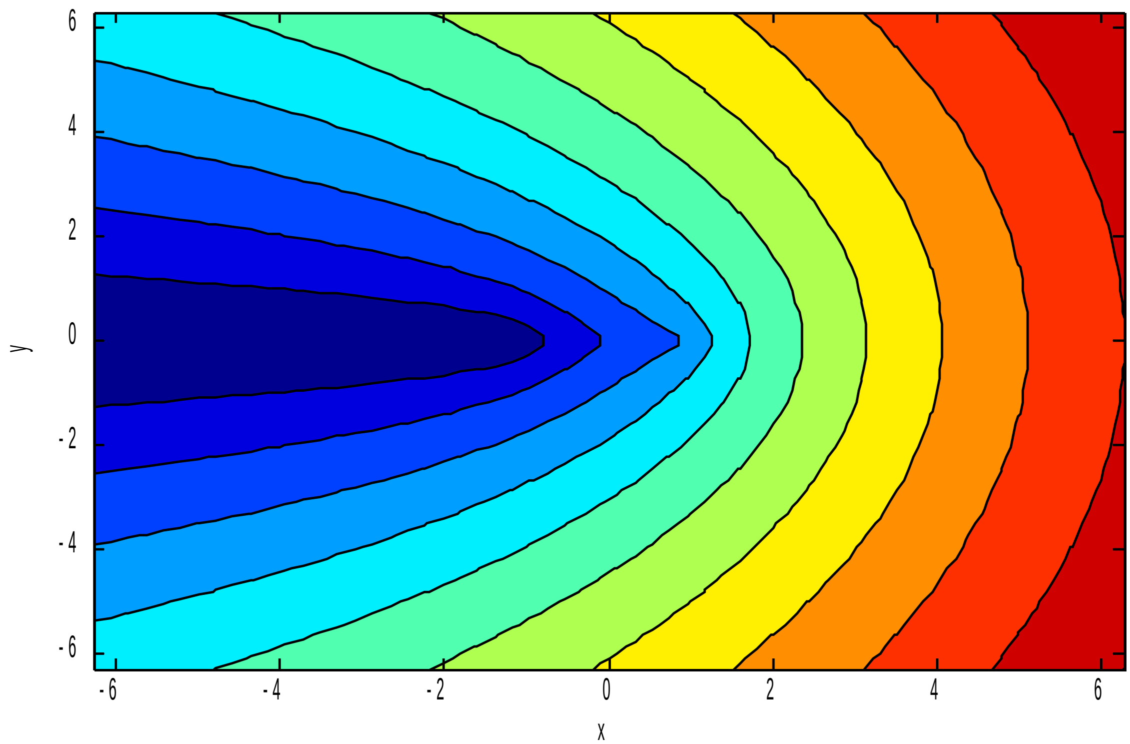




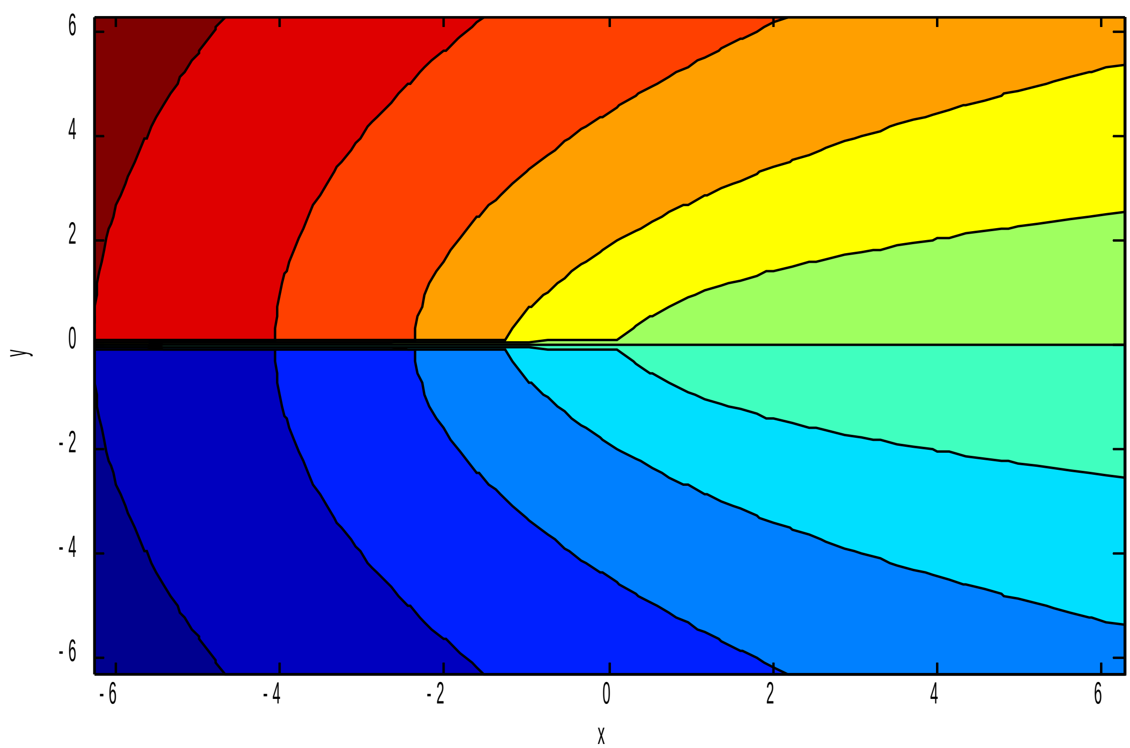
<u>NOT</u> possible



real(sqrt(complex(x,y)-1)+sqrt(complex(x,y)+1))



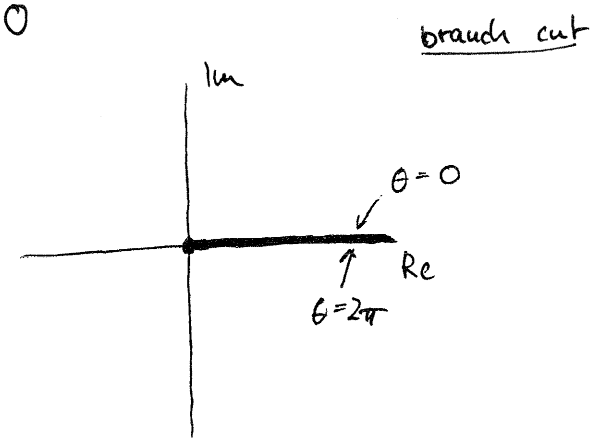
imag(sqrt(complex(x,y)-1)+sqrt(complex(x,y)+1))



```matlab
%% Problem 12.15 g matlab code
fr = @(x,y) real(sqrt(complex(x,y)-1) + sqrt(complex(x,y)+1));
fi = @(x,y) imag(sqrt(complex(x,y)-1) + sqrt(complex(x,y)+1));
subplot(211)
ezcontourf(fr)
subplot(212)
ezcontourf(fi)

% Note: Matlab uses a branch cut along the negative real axis.
```

12.20



The branch cut of the derivative f' also has to be from $z=0$ along the positive real axis, because f is not differentiable "across" the cut. Therefor f' ~~da~~ is cut in the same place.